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Research Article

Finite-Time Stability Analysis of Switched Genetic Regulatory Networks with Time-Varying Delays via Wirtinger's Integral Inequality

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The problem of finite-time stability of switched genetic regulatory networks (GRNs) with time-varying delays via Wirtinger's integral inequality is addressed in this study. A novel Lyapunov–Krasovskii functional is proposed to capture the dynamical characteristic of GRNs. Using Wirtinger's integral inequality, reciprocally convex combination technique and the average dwell time method conditions in the form of linear matrix inequalities (LMIs) are established for finite-time stability of switched GRNs. The applicability of the developed finite-time stability conditions is validated by numerical results.

1. Introduction

In recent years, GRNs have received much research attention, and many interesting results have been reported [1–8]. Generally, there are two types of gene network models, the Boolean model [9] and differential equation model [10]. In the Boolean model, the state converges to a terminal state via a series of state transitions that is determined by the Boolean rules. In this model, the activity of each gene is expressed in one of two states, ON or OFF, which is determined by a Boolean function by its own and by other related states. Whereas in the differential equation model, the variables describe the concentrations of gene products, such as mRNA and proteins as continuous values of the gene regulation system, and also, this model talks about the concentrations of gene products such as mRNA and proteins as variables in GRNs [11–14]. There are many research results on the stability analysis for GRNs with time delay (e.g., [15–18]).

Time delays are ubiquitous in many fields because of finite propagation speeds of signals, finite processing times, finite reaction times, and finite switching speed of amplifiers. Since the biological system especially GRNs is a slow process of transcription, translation, and translocation [19–23], the time delay cannot be avoided. From the long term investigations, time delay will bring instability of the system, sustained oscillations such as bifurcation [24–26]. So, it is of great importance to deal delayed GRNs. For instance, in [27], authors presented a different equation model for GRNs with constant time delays and proposed stability analysis for GRNs with time delays. In [28], authors developed delay dependent criteria for stability of GRNs with delay and free-weighting matrices.

On the other hand, Markovian switching and switched systems have been studied extensively over the past decades, for its capacity in modeling practical systems and its potential applications. Including a variety of subsystems as constituent parts, switched systems are governed by a switching rule to coordinate the switching. Recently, the stability problem of Markovian jump GRNs and switched GRNs has been investigated in [29–34]. As we all know, most of gene networks contain some kinds of switching mechanisms. For instance, by increasing stimulation or by changing some regulatory mechanisms, a bistable system can switch from one steady state to the other. In [33, 35–39], authors investigated the stability for switched systems with time delays by utilizing an average dwell time approach.

Recently, many kinds of finite-time issues have attracted particular research interests, and there have been some results on finite-time stabilization and synchronization [40–47]. However, to the best of the authors knowledge, there have been very few results on the finite-time stability problem for delayed GRNs with time delays [48, 49], and the purpose of this study is therefore to shorten such a gap.

Motivated by the above discussion, in this study, we are concerned with the finite-time stability of switched GRNs, where the parameter values switch from one mode to another. By utilizing the average dwell time approach and by using a novel Lyapunov–Krasovskii functional, it is shown that the finite-time stability problem is solvable if a set of linear matrix inequalities (LMIs) is feasible. Finally, three examples are provided in the end of the study to show the effectiveness of the proposed criteria.

The rest of this study is organized as follows: in Section 2, preliminaries and problem formulation are given. In Section 3, some conditions are established to ensure the finite-time stability of the considered system. In Section 4, three examples are illustrated to show the effectiveness of the obtained theoretical results. And finally, conclusions are given in Section 5.

Notations: throughout this study, $\mathbb{R}$, $\mathbb{R}^n$, and $\mathbb{R}^{n\times m}$ denote, respectively, the set of all real numbers, real n-dimensional space, and real $n\times m$-dimensional space. $\|\cdot\|$ denote the Euclidean norms in $\mathbb{R}^n$. For a vector or matrix P, P^T denotes its transpose. For a square matrix P, $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum eigenvalue and minimum eigenvalue of matrix P, respectively, and sym (P) is used to represent $P+P^T$. For simplicity, in symmetric block matrices, we often use $*$ to represent the term that is induced by symmetry.

2. Problem Description and Preliminaries

Consider the following nonlinear GRNs with time-varying delays described by

$$\begin{cases} \dot{e}_1(t) = -Ae_1(t) + Bf(e_2(t-\tau(t))) + I, \\ \dot{e}_2(t) = -Ce_2(t) + D(e_1(t-\sigma(t))), \end{cases} \tag{1}$$

where $e_1(t) = [e_{11}(t), e_{12}(t), \ldots, e_{1n}(t)]^T \in \mathbb{R}^n$, $e_2(t) = [e_{21}(t), e_{22}(t), \ldots, e_{2n}(t)]^T \in \mathbb{R}^n$:, $e_{2i}(t) \in \mathbb{R}$ are the concentrations of mRNA and protein, respectively; $f(\cdot) = [f_1(\cdot), \ldots, f_n(\cdot)]^T$ is the regulatory functions of mRNAs, $A = \operatorname{diag}(a_1, a_2, \ldots, a_n)$ and $C = \operatorname{diag}(c_1, c_2, \ldots, c_n)$ are the constant matrices, and they are the rates of degradation; $D = \operatorname{diag}(d_1, d_2, \ldots, d_n)$ represents the translation rate; $B = (b_{ij})$ is the regulative matrix, and $\tau(t)$ and $\sigma(t)$ are the time-varying delays.

For obtaining our conclusions, we make the following assumptions.

Assumption 1. f_s: $\mathbb{R} \longrightarrow \mathbb{R}$, $s=1, 2, \ldots, n$ are monotonically increasing functions with saturation and satisfy

$$0 \le \frac{f_s(a) - f_s(b)}{a-b} \le u_s, \quad \forall a, b \in \mathbb{R}, s = 1, 2, \ldots, n, \tag{2}$$

where u_s, $s=1, 2, \ldots, n$ are the nonnegative constants.

Assumption 2. $\tau(t)$ and $\sigma(t)$ are the time-varying delays satisfying

$$\begin{aligned} &0 \le \tau_1 \le \tau(t) \le \tau_2, \\ &\dot{\tau}(t) \le \tau_d < \infty, \\ &0 \le \sigma_1 \le \sigma(t) \le \sigma_2, \\ &\dot{\sigma}(t) \le \sigma_d < \infty, \\ &\tau_{12} = \tau_2 - \tau_1, \\ &\sigma_{12} = \sigma_2 - \sigma_1, \end{aligned} \tag{3}$$

where $\tau_1, \tau_2, \sigma_1, \sigma_2$ are the constants. The initial condition of system (1) is assumed to be

$$-\rho \le t \le 0, \rho = \max\{\tau_2, \sigma_2\}. \tag{4}$$

Use the following transformation:

$$\begin{aligned} x(t) &= e_1(t) - e_1(t)^*, \\ y(t) &= e_2(t) - e_2(t)^*, \end{aligned} \tag{5}$$

where $e_1^* = [e_{11}^*, e_{12}^*, \ldots, e_{1n}^*]^T$ and $e_2^* = [e_{21}^*, e_{22}^*, \ldots, e_{2n}^*]^T$ constitute an equilibrium point of system (1) and then shift the intended equilibrium point to the origin. In this way, the system equation turns to be

$$\begin{cases} \dot{x}(t) = -Ax(t) + Bg(y(t-\tau(t))), \\ \dot{y}(t) = -Cy(t) + D(x(t-\sigma(t))), \end{cases} \tag{6}$$

where $g(\cdot) = [g_1(\cdot), \ldots, g_n(\cdot)]^T$, and $g_s(y(t)) = f_s(y(t) + e_2^*) - f_s(e_2^*)$.

According to Assumption 1 and the definition of $g_s(\cdot)$, we know that $g_s(\cdot)$ is bounded, that is, $\exists F > 0$, such that $|g_s(\cdot)| \le F$, $s=1, 2, \ldots, n$ and satisfies the following sector condition:

$$0 \le \frac{g_s(a)}{a} \le u_s, \quad \forall a \in \mathbb{R}/\{0\}, s = 1, 2, \ldots, n. \tag{7}$$

Let $U = \operatorname{diag}\{u_1, u_2, \ldots, u_n\}$.

Sometimes, GRNs were described by the continuous time switched system, as in [33]; so system (6) can be described as the switching system with switching signal:

$$\begin{cases} \dot{x}(t) = -A_{p(t)}x(t) + B_{p(t)}g(y(t-\tau(t))), \\ \dot{y}(t) = -C_{p(t)}y(t) + D_{p(t)}(x(t-\sigma(t))), \end{cases} \tag{8}$$

where $p(t)$: $[0,\infty) \longrightarrow \mathbb{N} = \{1, 2, \ldots, N\}$ is the switching signal, which is a piece constant function depending on time t. For each $i \in \mathbb{N}$, the matrices are constant matrices of appropriate dimensions.

For the switching signal $p(t)$, we have the following switching sequence: $\{(i_0, t_0), \ldots, (i_k, t_k), \ldots, \mid i_k \in \mathbb{N}, k = 0, 1, \ldots\}$; in other words, when $t \in [t_k, t_{k+1})$, i_k^{th} subsystem is activated. To assume $x(t) = \phi(t)$, $y(t) = \psi(t)$.

For proving the theorem, we recall the following definition and lemmas.

Definition 1 (See [50]). The system (8) is said to be finite-time stable with respect to positive real numbers (c_1, c_2, T), if

$$\|\Phi(t)\|^2 + \|\Psi(t)\|^2 \le c_1 \Rightarrow \|x(t)\|^2 + \|y(t)\|^2 \le c_2, \quad t \in (0, T], \tag{9}$$

where

$$\begin{aligned} \|\Phi(t)\| &= \sup_{-\rho\le t\le 0}\left\{\|\phi(t)\|, \|\dot{\phi}(t)\|\right\}, \\ \|\Psi(t)\| &= \sup_{-\rho\le t\le 0}\left\{\|\psi(t)\|, \|\dot{\psi}(t)\|\right\}. \end{aligned} \tag{10}$$

Definition 2 (See [51]). For any $T_2 > T_1 \ge 0$, let $N_p(T_1, T_2)$ denote the switching number of $p(t)$ on an interval (T_1, T_2). If

$$N_p(T_1, T_2) \le N_0 + \frac{T_2 - T_1}{\tau_a} \tag{11}$$

holds for given $N_0 \ge 0$, $\tau_a > 0$, then the constant τ_a is called the average dwell time and N_0 is the chatter bound. Without loss of generality, we choose $N_0 = 0$ throughout this study.

Lemma 1 (See [52]). *For any constant matrix $M \in \mathbb{R}^{n\times n}$, $M = M^T > 0$, scalars $\eta_2 > \eta_1 > 0$, and vector function w: $[\eta_1, \eta_2] \longrightarrow \mathbb{R}^n$ such that the integrations concerned are well defined, and the following inequality holds*

$$\begin{aligned} \left[\int_{t-\eta_2}^{t-\eta_1} w(s)\mathrm{d}s\right]^T M\left[\int_{t-\eta_2}^{t-\eta_1} w(s)\mathrm{d}s\right] &\le (\eta_2 - \eta_1)\int_{t-\eta_2}^{t-\eta_1} w^T(s)Mw(s)\mathrm{d}s. \\ \left[\int_{-\eta_2}^{-\eta_1}\int_{t+\theta}^{t} w(s)\mathrm{d}s\right]^T M\left[\int_{-\eta_2}^{-\eta_1} w(s)\mathrm{d}s\right] &\le \frac{\left(\eta_2^2 - \eta_1^2\right)}{2}\int_{-\eta_2}^{-\eta_1}\int_{t+\theta}^{t} w^T(s)Mw(s)\mathrm{d}s. \end{aligned} \tag{12}$$

Lemma 2 (See [53]). *Let $f_1, f_2, \ldots, f_N$: $\mathbb{R}^m \longrightarrow \mathbb{R}$ have positive values in an open subset D of $\mathbb{R}^m$. Then, the reciprocally convex combination of f_i over D satisfies*

$$\min_{\left\{\beta_i \mid \beta_i > 0 \sum_i \beta_i = 1\right\}} \sum_i f_i(t) + \max_{g_{i,j}(t)} \sum_{i\neq j} g_{i,j}(t), \tag{13}$$

subjected to

$$\left\{g_{ij}: \mathbb{R}^m \longrightarrow \mathbb{R}, g_{j,i}(t) = g_{ij}(t), \begin{bmatrix} f_i(t) & g_{i,j}(t) \\ g_{i,j}(t) & f_j(t) \end{bmatrix} \ge 0\right\}. \tag{14}$$

Lemma 3 (See [54]). *For a positive definite matrix $M > 0$, the following inequality holds for all continuously differentiable function $x(t)$ in $[a, b] \in \mathbb{R}^{n\times n}$:*

$$\begin{aligned} -(b-a)\int_a^b \dot{x}^T(s)M\dot{x}(s)\mathrm{d}s &\le -\Phi_1^T M\Phi_1 - 3 - \Phi_2^T M\Phi_2, \\ &= -\begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix}^T \begin{bmatrix} M & 0 \\ 0 & 3M \end{bmatrix} \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix}, \end{aligned} \tag{15}$$

where $\Phi_1 = x(b) - x(a)$, and $\Phi_2 = x(b) + x(a) - 2/(b-a)\int_a^b x(s)ds$.

3. Main Results

In this section, we present a finite-time stability theorem for switching genetic regulatory networks with interval time-varying delays (8).

Theorem 1. *The switched genetic networks (8) is finite-time stable with respect to positive real numbers (c_1, c_2, T_f) and constants $\sigma_1, \sigma_2, \tau_1$, and τ_2; if there exist symmetric positive definite matrices P_{1i}, P_{2i}, Q_{ni} $(n = 1, 2, 3)$, R_{ni} $(n = 1, 2, \ldots, 12)$, S_{ni} $(n = 1, 2, \ldots, 4)$ for all $i \in \mathbb{N}$, the diagonal matrix $L_m = diag(l_{1m}, l_{2m}, \ldots, l_{nm}) \ge 0$, $m = 1, 2$, and positive scalars $\mu \ge 1$ and $\alpha_i = \alpha > 0$ such that the following LMIs hold*

$$\begin{aligned} &\begin{bmatrix} R_{3i} & M_{1i} \\ * & R_{4i} \end{bmatrix} > 0, \\ &\begin{bmatrix} R_{6i} & \overline{R}_{6i} \\ * & R_{6i} \end{bmatrix} \ge 0, \\ &\begin{bmatrix} R_{8i} & \overline{R}_{8i} \\ * & R_{8i} \end{bmatrix} \ge 0, \\ &\begin{bmatrix} R_{10i} & \overline{R}_{10i} \\ * & R_{10i} \end{bmatrix} > 0, \\ &\begin{bmatrix} R_{12i} & M_{12i} \\ * & R_{12i} \end{bmatrix} > 0, \end{aligned} \tag{16}$$

$$\Theta_{p(t)} = \left[\Theta_{ij}\right]_{22\times 22} < 0, \tag{17}$$

$$\begin{aligned} P_{li} &\leq \mu P_{lj}, \quad l = 1, 2, \\ Q_{li} &\leq \mu Q_{lj}, \quad l = 1, 2, 3, \\ R_{li} &\leq \mu R_{lj}, \quad l = 1, 2, \ldots, 12, \\ M_{1i} &\leq \mu M_{1j}, S_{li} \leq \mu S_{lj}, \quad l = 1, 2, 3, \end{aligned} \tag{18}$$

$$e^{\left\{\sum_{i=1}^{N} \alpha_i T\right\}} dc_1 \leq c_2 (\lambda_1 + \lambda_2), \tag{19}$$

and the average dwell time of the switching signal $p(t)$ satisfies

$$\tau_{ai} > \tau_{ai}^* = \frac{T \ln \mu_i}{\ln (c_2 (\lambda_1 + \lambda_2)) - \ln\left(e^{\alpha_i T} dc_1\right)}, \tag{20}$$

where

$$\Theta_{11} = -P_{1i}A - A^T P_{1i}^T + Q_{1i} + Q_{3i} + \sigma_1^2 R_{5i} + \sigma_{12}^2 R_{6i} - 3e^{-\alpha\sigma_1} R_{7i} - e^{-\alpha\sigma_1} R_{7i} + \alpha P_{1i},$$

$$\Theta_{13} = e^{-\alpha\sigma_1} R_{7i} - 3e^{-\alpha\sigma_1} R_{7i},$$

$$\Theta_{110} = P_{1i}B, \Theta_{113} = \frac{6}{\sigma_1} e^{-\alpha\sigma_1} R_{7i},$$

$$\Theta_{22} = R_{1i} + R_{3i} - P_{2i}C - C^T P_{2i}^T + \tau_1^2 R_{9i} + \tau_{12}^2 R_{10i} - e^{-\alpha\tau_1} R_{11i} - 3e^{-\alpha\tau_1} R_{11i} + \alpha P_{2i},$$

$$\Theta_{24} = P_{2i}D,$$

$$\Theta_{26} = e^{-\alpha\tau_1} R_{11i} - 3e^{-\alpha\tau_1} R_{11i},$$

$$\Theta_{29} = M_{1i} + U^T L_1^T,$$

$$\Theta_{218} = \frac{6}{\tau_1} e^{-\alpha\tau_1} R_{11i},$$

$$\Theta_{33} = -e^{-\alpha\sigma_1} Q_{1i} + e^{-\alpha\sigma_1} Q_{2i} - e^{-\alpha\sigma_1} R_{7i} - 3e^{-\alpha\sigma_1} R_{7i} - e^{-\alpha\sigma_2} R_{8i},$$

$$\Theta_{34} = -e^{-\alpha\sigma_2} \overline{R}_{8i}^T + e^{-\alpha\sigma_2} R_{8i},$$

$$\Theta_{35} = e^{-\alpha\sigma_2} \overline{R}_{8i}^T, \Theta_{313} = \frac{6}{\sigma_1} e^{-\alpha\sigma_1} R_{7i},$$

$$\Theta_{44} = -(1 - \sigma_d) e^{-\alpha\sigma(t)} Q_{3i} + 2e^{-\alpha\sigma_2} \overline{R}_{8i} - 2e^{-\alpha\sigma_2} R_{8i},$$

$$\Theta_{45} = e^{-\alpha\sigma_2} \overline{R}_{8i}^T + e^{-\alpha\sigma_2} R_{8i},$$

$$\Theta_{55} = e^{-\alpha\sigma_2} Q_{2i}^T - e^{-\alpha\sigma_2} R_{8i},$$

$$\Theta_{66} = -e^{-\alpha\tau_1} R_{1i} + e^{-\alpha\tau_1} R_{2i} - e^{-\alpha\tau_1} R_{11i} - 3e^{-\alpha\tau_1} R_{11i} - e^{-\alpha\tau_2} R_{12i},$$

$$\Theta_{67} = -e^{-\alpha\tau_2} \overline{R}_{12i}^T + e^{-\alpha\tau_2} R_{12i},$$

$$\Theta_{68} = -e^{-\alpha\tau_2} \overline{R}_{12i}^T,$$

$$\Theta_{618} = \frac{6}{\tau_1} e^{-\alpha\tau_1} R_{11i},$$

$$\Theta_{77} = -(1 - \tau_d) e^{-\alpha\tau(t)} R_{3i} + 2e^{-\alpha\tau_2} \overline{R}_{12i} - 2e^{-\alpha\tau_2} R_{12i},$$

$$\Theta_{78} = e^{-\alpha\tau_2} \overline{R}_{12i}^T + e^{-\alpha\tau_2} R_{12i},$$

$$\Theta_{710} = -(1 - \tau_d) e^{-\alpha\tau(t)} M_{1i} + UL_2,$$

$$
\begin{aligned}
\Theta_{88} &= -e^{-\alpha\tau_2}R_{2i}^T - e^{-\alpha\tau_2}R_{12i},\\
\Theta_{99} &= R_{4i} - 2L_1,\\
\Theta_{1010} &= -(1-\tau_d)e^{-\alpha\tau_2}R_{4i} - 2L_2,\\
\Theta_{1111} &= \sigma_1^2 R_{7i} + \sigma_{12}^2 R_{8i} + e^{\alpha t}\frac{e^{\alpha\sigma_2} - \alpha\sigma_2 - 1}{\alpha^2}S_{1i} + e^{\alpha t}\frac{e^{\alpha\sigma_2} - e^{\alpha\sigma_1} - \alpha(\sigma_2 - \sigma_1)}{\alpha^2}S_{2i},\\
\Theta_{1212} &= \tau_1^2 R_{11i} + \tau_{12}^2 R_{12i} + e^{\alpha t}\frac{e^{\alpha\tau_2} - \alpha\tau_2 - 1}{\alpha^2}S_{3i} + e^{\alpha t}\frac{e^{\alpha\tau_2} - e^{\alpha\tau_1} - \alpha(\tau_2 - \tau_1)}{\alpha^2}S_{4i},\\
\Theta_{1313} &= -e^{\alpha\sigma_1}R_{5i} - \frac{12}{\sigma_1^2}e^{-\alpha\sigma_1}R_{7i}, \Theta_{1414} = -e^{\alpha\sigma_2}R_{6i},\\
\Theta_{1415} &= -e^{\alpha\sigma_2}\overline{R}_{6i}, \Theta_{1515} = -e^{\alpha\sigma_2}R_{6i},\\
\Theta_{1616} &= -e^{\alpha t}S_{1i},\\
\Theta_{1717} &= -e^{\alpha t}S_{2i},\\
\Theta_{1818} &= -e^{\alpha\tau_1}R_{9i} - \frac{12}{\tau_1^2}e^{-\alpha\tau_1}R_{11i},\\
\Theta_{1919} &= -e^{\alpha\tau_2}R_{10i}, \Theta_{1920} = -e^{\alpha\tau_2}\overline{R}_{10i},\\
\Theta_{2020} &= -e^{\alpha\tau_2}R_{10i}, \Theta_{2121} = -e^{\alpha t}S_{3i},\\
\Theta_{2222} &= -e^{\alpha t}S_{4i},\\
d &= \lambda_3 + \lambda_4 + \lambda_5\sigma_1 e^{-\alpha\sigma_1} + \lambda_6(\sigma_2 - \sigma_1)e^{-\alpha\sigma_2} + \lambda_7\sigma_2 e^{-\alpha\sigma_2} + \lambda_8\tau_1 e^{-\alpha\tau_1}\\
&\quad + \lambda_9(\tau_2 - \tau_1)e^{-\alpha\tau_1} + \lambda_{10}\tau_2 e^{-\alpha\tau_2} + 2\lambda_{11}\tau_2 e^{-\alpha\tau_2} + \lambda_{12}\tau_2 e^{-\alpha\tau_2} + \lambda_{13}\frac{\sigma_1^3}{2}e^{-\alpha\sigma_1}\\
&\quad + \lambda_{14}\frac{(\sigma_2 - \sigma_1)^3}{2}e^{-\alpha\sigma_2} + \lambda_{15}\frac{\sigma_1^3}{2}e^{-\alpha\sigma_1} + \lambda_{16}\frac{(\tau_2 - \tau_1)^3}{2}e^{-\alpha\tau_2} + \lambda_{17}\frac{\tau_1^3}{2}e^{-\alpha\tau_1}\\
&\quad + \lambda_{18}\frac{(\tau_2 - \tau_1)^3}{2}e^{-\alpha\tau_2} + \lambda_{19}\frac{\tau_1^3}{2}e^{-\alpha\tau_1} + \lambda_{20}\frac{(\tau_2 - \tau_1)^3}{2}e^{-\alpha\tau_2} + \lambda_{21}\frac{\sigma_1^5}{4}e^{-\alpha\sigma_1}\\
&\quad + \lambda_{22}\frac{(\sigma_2^2 - \sigma_1^2)(\sigma_2 - \sigma_1)^3}{4}e^{-\alpha\sigma_2} + \lambda_{23}\frac{\tau_1^5}{4}e^{-\alpha\tau_1} + \lambda_{24}\frac{(\tau_2^2 - \tau_1^2)(\tau_2 - \tau_1)^3}{4}e^{-\alpha\tau_2},\\
\lambda_1 &= \min_{i\in\mathbb{N}}\lambda_{\min}(P_{1i}), \lambda_2 = \min_{i\in\mathbb{N}}\lambda_{\min}(P_{2i}), \lambda_3 = \max_{i\in\mathbb{N}}\lambda_{\max}(P_{1i}),\\
\lambda_4 &= \max_{i\in\mathbb{N}}\lambda_{\max}(P_{2i}), \lambda_5 = \max_{i\in\mathbb{N}}\lambda_{\max}(Q_{1i}), \lambda_6 = \max_{i\in\mathbb{N}}\lambda_{\max}(Q_{2i}),\\
\lambda_7 &= \max_{i\in\mathbb{N}}\lambda_{\max}(Q_{3i}), \lambda_8 = \max_{i\in\mathbb{N}}\lambda_{\max}(R_{1i}), \lambda_{10} = \max_{i\in\mathbb{N}}\lambda_{\max}(R_{2i}),\\
\lambda_{11} &= \max_{i\in\mathbb{N}}\lambda_{\max}(R_{3i}), \lambda_{12} = \max_{i\in\mathbb{N}}\lambda_{\max}(R_{4i}), \lambda_{13} = \max_{i\in\mathbb{N}}\lambda_{\max}(R_{5i}),\\
\lambda_{14} &= \max_{i\in\mathbb{N}}\lambda_{\max}(R_{6i}), \lambda_{15} = \max_{i\in\mathbb{N}}\lambda_{\max}(R_{7i}), \lambda_{16} = \max_{i\in\mathbb{N}}\lambda_{\max}(R_{8i}),\\
\lambda_{17} &= \max_{i\in\mathbb{N}}\lambda_{\max}(R_{9i}), \lambda_{18} = \max_{i\in\mathbb{N}}\lambda_{\max}(R_{10i}), \lambda_{19} = \max_{i\in\mathbb{N}}\lambda_{\max}(R_{11i}),\\
\lambda_{20} &= \max_{i\in\mathbb{N}}\lambda_{\max}(R_{12i}), \lambda_{21} = \max_{i\in\mathbb{N}}\lambda_{\max}(S_{1i}), \lambda_{22} = \max_{i\in\mathbb{N}}\lambda_{\max}(S_{2i}),\\
\lambda_{23} &= \max_{i\in\mathbb{N}}\lambda_{\max}(S_{3i}), \lambda_{24} = \max_{i\in\mathbb{N}}\lambda_{\max}(S_{4i}).
\end{aligned}
\tag{21}
$$

Proof. Choose the Lyapunov functional candidate as

$$V_{p(t)}(t) = V_{1p(t)} + V_{2p(t)} + V_{3p(t)} + V_{4p(t)} + V_{5p(t)} + V_{6p(t)}, \tag{22}$$

where

$$\begin{aligned}
V_{1p(t)} &= x^T(t)P_{1i}x(t) + y^T(t)P_{2i}y(t),\\
V_{2p(t)} &= \int_{t-\sigma_1}^{t} e^{\alpha(s-t)}x^T(s)Q_{1i}x(s)\mathrm{d}s + \int_{t-\sigma_2}^{t-\sigma_1} e^{\alpha(s-t)}x^T(s)Q_{2i}x(s)\mathrm{d}s\\
&\quad + \int_{t-\sigma(t)}^{t} e^{\alpha(s-t)}x^T(s)Q_{3i}x(s)\mathrm{d}s,\\
V_{3p(t)} &= \int_{t-\tau_1}^{t} e^{\alpha(s-t)}y^T(s)R_{1i}y(s)\mathrm{d}s + \int_{t-\tau_2}^{t-\tau_1} e^{\alpha(s-t)}y^T(s)R_{2i}y(s)\mathrm{d}s\\
&\quad + \int_{t-\tau(t)}^{t} e^{\alpha(s-t)}\begin{bmatrix} y(s)\\ g(y(s))\end{bmatrix}^T\begin{bmatrix} R_{3i} & M_{1i}\\ * & R_{4i}\end{bmatrix}\begin{bmatrix} y(s)\\ g(y(s))\end{bmatrix}\mathrm{d}s,\\
V_{4p(t)} &= \int_{-\sigma_1}^{0}\int_{t+\theta}^{t} \sigma_1 e^{\alpha(s-t)}x^T(s)R_{5i}x(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\sigma_2}^{-\sigma_1}\int_{t+\theta}^{t} \sigma_{12}e^{\alpha(s-t)}x^T(s)R_{6i}x(s)\mathrm{d}s\mathrm{d}\theta\\
&\quad + \int_{-\sigma_1}^{0}\int_{t+\theta}^{t} \sigma_1 e^{\alpha(s-t)}\dot{x}^T(s)R_{7i}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\sigma_2}^{-\sigma_1}\int_{t+\theta}^{t} \sigma_{12}e^{\alpha(s-t)}\dot{x}^T(s)R_{8i}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta,\\
V_{5p(t)} &= \int_{-\tau_1}^{0}\int_{t+\theta}^{t} \tau_1 e^{\alpha(s-t)}y^T(s)R_{9i}y(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} \tau_{12}e^{\alpha(s-t)}y^T(s)R_{10i}y(s)\mathrm{d}s\mathrm{d}\theta\\
&\quad + \int_{-\tau_1}^{0}\int_{t+\theta}^{t} \tau_1 e^{\alpha(s-t)}\dot{y}^T(s)R_{11i}\dot{y}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} \tau_{12}e^{\alpha(s-t)}\dot{y}^T(s)R_{12i}\dot{y}(s)\mathrm{d}s\mathrm{d}\theta,\\
V_{6p(t)} &= \int_{-\sigma_2}^{0}\int_{\theta}^{0}\int_{t+\nu}^{t} \frac{\sigma_1^2}{2}e^{\alpha(s-\theta)}\dot{x}^T(s)S_{1i}\dot{x}(s)\mathrm{d}s\mathrm{d}\nu\mathrm{d}\theta + \int_{-\sigma_2}^{-\sigma_1}\int_{\theta}^{0}\int_{t+\nu}^{t} \frac{1}{2}\left(\sigma_2^2-\sigma_1^2\right)e^{\alpha(s-\theta)}\dot{x}^T(s)S_{2i}\dot{x}(s)\mathrm{d}s\mathrm{d}\nu\mathrm{d}\theta\\
&\quad + \int_{-\tau_2}^{0}\int_{\theta}^{0}\int_{t+\nu}^{t} \frac{\tau_1^2}{2}e^{\alpha(s-\theta)}\dot{y}^T(s)S_{3i}\dot{y}(s)\mathrm{d}s\mathrm{d}\nu\, d\theta + \int_{-\tau_2}^{-\tau_1}\int_{\theta}^{0}\int_{t+\nu}^{t} \frac{1}{2}\left(\tau_2^2-\tau_1^2\right)e^{\alpha(s-\theta)}\dot{y}^T(s)S_{4i}\dot{y}(s)\mathrm{d}s\mathrm{d}\nu\mathrm{d}\theta.
\end{aligned} \tag{23}$$

Taking the derivatives of $V_{p(t)}$ along the trajectory of system (8), we have that

$$\dot{V}_{1p(t)} = 2x^T(t)P_{1i}\dot{x}(t) + 2y^T(t)P_{2i}\dot{y}(t), \tag{24}$$

$$\begin{aligned}
\dot{V}_{2p(t)} = -\alpha V_{2p(t)} &+ x^T(t)Q_{1i}x(t) - e^{-\alpha\sigma_1}x^T(t-\sigma_1)Q_{1i}x(t-\sigma_1)\\
&+ e^{-\alpha\sigma_1}x^T(t-\sigma_1)Q_{2i}x(t-\sigma_1) - e^{-\alpha\sigma_2}x^T(t-\sigma_2)Q_{2i}x(t-\sigma_2)\\
&+ x^T(t)Q_{3i}x(t) - (1-\dot{\sigma}(t))e^{-\alpha\sigma(t)}x^T(t-\sigma(t))Q_{3i}x(t-\sigma(t)),
\end{aligned} \tag{25}$$

$$\begin{aligned}
\dot{V}_{3p(t)} &= -\alpha V_{3p(t)} + y^T(t)R_{1i}y(t) - e^{-\alpha\tau_1}y^T(t-\tau_1)R_{1i}y(t-\tau_1) + e^{-\alpha\tau_1}y^T(t-\tau_1)R_{2i}y(t-\tau_1) \\
&\quad - e^{-\alpha\tau_2}y^T(t-\tau_2)R_{2i}y(t-\tau_2) + \begin{bmatrix} y(t) \\ g(y(t)) \end{bmatrix}^T \begin{bmatrix} R_{3i} & M_{1i} \\ * & R_{4i} \end{bmatrix} \begin{bmatrix} y(t) \\ g(y(t)) \end{bmatrix} \\
&\quad - (1-\dot{\tau}(t))e^{-\alpha\tau(t)} \begin{bmatrix} y(t-\tau(t)) \\ g(y(t-\tau(t))) \end{bmatrix}^T \begin{bmatrix} R_{3i} & M_{1i} \\ * & R_{4i} \end{bmatrix} \begin{bmatrix} y(t-\tau(t)) \\ g(y(t-\tau(t))) \end{bmatrix} \\
&\leq -\alpha V_{3p(t)} + y^T(t)R_{1i}y(t) - e^{-\alpha\tau_1}y^T(t-\tau_1)R_{1i}y(t-\tau_1) + e^{-\alpha\tau_1}y^T(t-\tau_1)R_{2i}y(t-\tau_1) \\
&\quad - e^{-\alpha\tau_2}y^T(t-\tau_2)R_{2i}y(t-\tau_2) + \begin{bmatrix} y(t) \\ g(y(t)) \end{bmatrix}^T \begin{bmatrix} R_{3i} & M_{1i} \\ * & R_{4i} \end{bmatrix} \begin{bmatrix} y(t) \\ g(y(t)) \end{bmatrix} \\
&\quad - (1-\dot{\tau}(t))e^{-\alpha\tau_2} \begin{bmatrix} y(t-\tau(t)) \\ g(y(t-\tau(t))) \end{bmatrix}^T \begin{bmatrix} R_{3i} & M_{1i} \\ * & R_{4i} \end{bmatrix} \begin{bmatrix} y(t-\tau(t)) \\ g(y(t-\tau(t))) \end{bmatrix},
\end{aligned} \tag{26}$$

$$\begin{aligned}
\dot{V}_{4p(t)} &= -\alpha V_{4p(t)} + \sigma_1^2 x^T(t)R_{5i}x(t) - \int_{t-\sigma_1}^{t} \sigma_1 e^{\alpha(s-t)}x^T(s)R_{5i}x(s)\mathrm{d}s \\
&\quad + \sigma_{12}^2 x^T(t)R_{6i}x(t) - \int_{t-\sigma_2}^{t-\sigma_1} \sigma_{12} e^{\alpha(s-t)}x^T(s)R_{6i}x(s)\mathrm{d}s \\
&\quad + \sigma_1^2 \dot{x}^T(t)R_{7i}\dot{x}(t) - \int_{t-\sigma_1}^{t} \sigma_1 e^{\alpha(s-t)}\dot{x}^T(s)R_{7i}\dot{x}(s)\mathrm{d}s \\
&\quad + \sigma_{12}^2 \dot{x}^T(t)R_{8i}\dot{x}(t) - \int_{t-\sigma_2}^{t-\sigma_1} \sigma_{12} e^{\alpha(s-t)}\dot{x}^T(s)R_{8i}\dot{x}(s)\mathrm{d}s,
\end{aligned} \tag{27}$$

$$\begin{aligned}
\dot{V}_{5p(t)} &= -\alpha V_{5p(t)} + \tau_1^2 y^T(t)R_{9i}y(t) - \int_{t-\tau_1}^{t} \tau_1 e^{\alpha(s-t)}y^T(s)R_{9i}y(s)\mathrm{d}s \\
&\quad + \tau_{12}^2 y^T(t)R_{10i}y(t) - \int_{t-\tau_2}^{t-\tau_1} \tau_{12} e^{\alpha(s-t)}y^T(s)R_{10i}y(s)\mathrm{d}s \\
&\quad + \tau_1^2 \dot{y}^T(t)R_{11i}\dot{y}(t) - \int_{t-\tau_1}^{t} \tau_1 e^{\alpha(s-t)}\dot{y}^T(s)R_{11i}\dot{y}(s)\mathrm{d}s \\
&\quad + \tau_{12}^2 \dot{y}^T(t)R_{12i}\dot{y}(t) - \int_{t-\tau_2}^{t-\tau_1} \tau_{12} e^{\alpha(s-t)}\dot{y}^T(s)R_{12i}\dot{y}(s)\mathrm{d}s,
\end{aligned} \tag{28}$$

$$\begin{aligned}
\dot{V}_{6p(t)} &= -\alpha V_{6p(t)} + e^{\alpha t}\dot{x}^T(t)\frac{e^{\alpha\sigma_2} - \alpha\sigma_2 - 1}{\alpha^2} S_{1i}\dot{x}(t) - e^{\alpha t}\int_{-\sigma_2}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)S_{1i}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \\
&\quad + e^{\alpha t}\dot{x}^T(t)\frac{e^{\alpha\sigma_2} - e^{\alpha\sigma_1} - \alpha(\sigma_2-\sigma_1)}{\alpha^2} S_{2i}\dot{x}(t) - e^{\alpha t}\int_{-\sigma_2}^{-\sigma_1}\int_{t+\theta}^{t} \dot{x}^T(s)S_{2i}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \\
&\quad + e^{\alpha t}\dot{x}^T(t)\frac{e^{\alpha\tau_2} - \alpha\tau_2 - 1}{\alpha^2} S_{3i}\dot{x}(t) - e^{\alpha t}\int_{-\tau_2}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)S_{3i}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \\
&\quad + e^{\alpha t}\dot{x}^T(t)\frac{e^{\alpha\tau_2} - e^{\alpha\tau_1} - \alpha(\tau_2-\tau_1)}{\alpha^2} S_{4i}\dot{x}(t) - e^{\alpha t}\int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} \dot{x}^T(s)S_{4i}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta.
\end{aligned} \tag{29}$$

From Lemmas 1 and 3, we have

$$
\begin{aligned}
&-\sigma_1 \int_{t-\sigma_1}^{t} e^{\alpha(s-t)} x^T(s) R_{5i} x(s) \mathrm{d}s \leq -e^{-\alpha\sigma_1} \left(\int_{t-\sigma_1}^{t} x(s) \mathrm{d}s \right)^T R_{5i} \left(\int_{t-\sigma_1}^{t} x(s) \mathrm{d}s \right), \\
&-\sigma_1 \int_{t-\sigma_1}^{t} e^{\alpha(s-t)} \dot{x}^T(s) R_{7i} \dot{x}(s) \mathrm{d}s \leq -e^{-\alpha\sigma_1} \begin{bmatrix} \Phi_1(t) \\ \Phi_2(t) \end{bmatrix}^T \begin{bmatrix} R_{7i} & 0 \\ * & 3R_{7i} \end{bmatrix} \begin{bmatrix} \Phi_1(t) \\ \Phi_2(t) \end{bmatrix}, \\
&-\tau_1 \int_{t-\tau_1}^{t} e^{\alpha(s-t)} y^T(s) R_{9i} y(s) \mathrm{d}s \leq -e^{-\alpha\tau_1} \left(\int_{t-\tau_1}^{t} y(s) \mathrm{d}s \right)^T R_{9i} \left(\int_{t-\tau_1}^{t} y(s) \mathrm{d}s \right), \\
&-\tau_1 \int_{t-\tau_1}^{t} e^{\alpha(s-t)} \dot{y}^T(s) R_{11i} \dot{y}(s) \mathrm{d}s \leq e^{-\alpha\tau_1} \begin{bmatrix} \Phi_3(t) \\ \Phi_4(t) \end{bmatrix}^T \begin{bmatrix} R_{11i} & 0 \\ * & 3R_{11i} \end{bmatrix} \begin{bmatrix} \Phi_3(t) \\ \Phi_4(t) \end{bmatrix},
\end{aligned}
\tag{30}
$$

and from Lemma 2, we can obtain

$$
\begin{aligned}
&-\sigma_{12} \int_{t-\sigma_2}^{t-\sigma_1} e^{\alpha(s-t)} x^T(s) R_{6i} x(s) \mathrm{d}s = -\sigma_{12} e^{-\alpha\sigma_2} \int_{t-\sigma_2}^{t-\sigma(t)} x^T(s) R_{6i} x(s) \mathrm{d}s \\
&-\sigma_{12} e^{-\alpha\sigma_2} \int_{t-\sigma(t)}^{t-\sigma_1} x^T(s) R_{6i} x(s) \mathrm{d}s \leq e^{-\alpha\sigma_2} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}^T \begin{bmatrix} -R_{6i} & \overline{R}_{6i} \\ * & -R_{6i} \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}.
\end{aligned}
\tag{31}
$$

Similarly, we have

$$
-\sigma_{12} \int_{t-\sigma_2}^{t-\sigma_1} e^{\alpha(s-t)} x^T(s) R_{8i} x(s) \mathrm{d}s \leq e^{-\alpha\sigma_2} \begin{bmatrix} \psi_3 \\ \psi_4 \end{bmatrix}^T \begin{bmatrix} -R_{8i} & \overline{R}_{8i} \\ * & -R_{8i} \end{bmatrix} \begin{bmatrix} \psi_3 \\ \psi_4 \end{bmatrix},
\tag{32}
$$

$$
\begin{aligned}
&-\tau_{12} \int_{t-\tau_2}^{t-\tau_1} e^{\alpha(s-t)} x^T(s) R_{10i} x(s) \mathrm{d}s = -\tau_{12} e^{-\alpha\tau_2} \int_{t-\tau_2}^{t-\tau(t)} x^T(s) R_{10i} x(s) \mathrm{d}s \\
&-\tau_{12} e^{-\alpha\tau_2} \int_{t-\tau(t)}^{t-\tau_1} x^T(s) R_{10i} x(s) \mathrm{d}s \leq e^{-\alpha\tau_2} \begin{bmatrix} \psi_5 \\ \psi_6 \end{bmatrix}^T \begin{bmatrix} -R_{10i} & \overline{R}_{10i} \\ * & -R_{10i} \end{bmatrix} \begin{bmatrix} \psi_5 \\ \psi_6 \end{bmatrix},
\end{aligned}
\tag{33}
$$

$$
-\tau_{12} \int_{t-\tau_2}^{t-\tau_1} e^{\tau(s-t)} x^T(s) R_{12i} x(s) \mathrm{d}s \leq e^{-\alpha\tau_2} \begin{bmatrix} \psi_7 \\ \psi_8 \end{bmatrix}^T \begin{bmatrix} -R_{12i} & \overline{R}_{12i} \\ * & -R_{12i} \end{bmatrix} \begin{bmatrix} \psi_7 \\ \psi_8 \end{bmatrix},
\tag{34}
$$

where

$$
\begin{aligned}
\psi_1 &= \int_{t-\sigma_2}^{t-\sigma(t)} x(s)\mathrm{d}s,\\
\psi_2 &= \int_{t-\sigma(t)}^{t-\sigma_1} x(s)\mathrm{d}s,\\
\psi_3 &= x(t-\sigma(t)) - x(t-\sigma_2),\\
\psi_4 &= x(t-\sigma_1) - x(t-\sigma(t)),\\
\psi_5 &= \int_{t-\tau_2}^{t-\tau(t)} y(s)\mathrm{d}s,\\
\psi_6 &= \int_{t-\tau(t)}^{t-\tau_1} y(s)\mathrm{d}s,\\
\psi_7 &= y(t-\tau(t)) - y(t-\tau_2),\\
\psi_8 &= y(t-\tau_1) - y(t-\tau(t)),\\
\Phi_1(t) &= x(t) - x(t-\sigma_1),\\
\Phi_2(t) &= x(t) + x(t-\sigma_1) - \frac{2}{\sigma_1}\int_{t-\sigma_1}^{t} x(s)\mathrm{d}s,\\
\Phi_3(t) &= y(t) - y(t-\tau_1),\\
\Phi_4(t) &= y(t) + y(t-\tau_1) - \frac{2}{\tau_1}\int_{t-\tau_1}^{t} y(s)\mathrm{d}s.
\end{aligned}
\tag{35}
$$

Meanwhile, for any $L_m = \mathrm{diag}(l_{1m}, l_{2m}, \ldots, l_{nm}) \geq 0, m = 1, 2$, the following inequality is true from Assumption 1.

$$
\begin{aligned}
&-2\sum_{i=1}^{n} l_{1i} g_i(y_i(t))[g_i(y_i(t)) - u_i y_i(t)]\\
&-2\sum_{i=1}^{n} l_{2i} g_i(y_i(t-\tau(t)))[g_i(y_i(t-\tau(t))) - u_i y_i(t-\tau(t))] \geq 0.
\end{aligned}
\tag{36}
$$

It can written as

$$
\begin{aligned}
&-2gT(y(t))L_1 g(y(t)) + 2y^T(t)UL_1 g(y(t))\\
&-2g^T(y(t-\tau(t)))L_2 g(y(t-\tau(t)))\\
&+2y^T(t-\tau(t))UL_2 g(y(t-\tau(t))) \geq 0.
\end{aligned}
\tag{37}
$$

What is more, the following equations are true for any matrices N_1, N_2 with appropriate dimensions from system (8).

$$
2\dot{x}^T(t)N_1[-Ax(t) + Bg(y(t-\tau(t))) - \dot{x}(t)] = 0, \tag{38}
$$

$$
2\dot{y}^T(t)N_2[-Cy(t) + Dx(t-\tau(t)) - \dot{y}(t)] = 0. \tag{39}
$$

From (24) to (39), we have that

$$
\dot{V}_{p(t)} - \alpha V_{p(t)} \leq \xi^T(t)\Theta_i \xi(t), \tag{40}
$$

where

$$
\begin{aligned}
\xi^T(t) = \Big[& x^T(t) \quad y^T(t) \quad x^T(t-\sigma_1) \quad x^T(t-\sigma(t)) \quad x^T(t-\sigma_2) \quad y^T(t-\tau_1)\\
& y^T(t-\tau(t)) \quad y^T(t-\tau_2) \quad g^T(y(t)) \quad g^T(y(t-\tau(t))) \quad \dot{x}^T(t) \quad \dot{y}^T(t) \quad \int_{t-\sigma_1}^{t} x^T(s)\mathrm{d}s\\
& \int_{t-\sigma_2}^{t-\sigma(t)} x^T(s)\mathrm{d}s \quad \int_{t-\sigma(t)}^{t-\sigma_1} x^T(s)\mathrm{d}s \quad \int_{t-\sigma_1}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)\mathrm{d}s\mathrm{d}\theta \quad \int_{-\sigma_2}^{-\sigma_1}\int_{t+\theta}^{t} \dot{x}^T(s)\mathrm{d}s\mathrm{d}\theta\\
& \int_{t-\tau_1}^{t} y^T(s)\mathrm{d}s \quad \int_{t-\tau_2}^{t-\tau(t)} y^T(s)\mathrm{d}s \quad \int_{t-\tau(t)}^{t-\tau_1} y^T(s)\mathrm{d}s \quad \int_{t-\tau_1}^{0}\int_{t+\theta}^{t} \dot{y}^T(s)\mathrm{d}s\mathrm{d}\theta\\
& \int_{-\tau_2}^{-\tau_1}\int_{t+\theta}^{t} \dot{y}^T(s)\mathrm{d}s\mathrm{d}\theta \Big].
\end{aligned}
\tag{41}
$$

By condition (17), we have

$$
\dot{V}_{p(t)} + \alpha V_{p(t)} < 0. \tag{42}
$$

Note that

$$
\frac{d}{\mathrm{d}t}\left(e^{-\alpha t}V_{p(t)}\right) < 0. \tag{43}
$$

For any $T > 0$, let $t_0 = 0$, and we denote $t_0, t_1, t_2, t_3, \ldots, t_i, \ldots, t_{N_p(0,T)}$ and $t_1, t_2, \ldots, t_i, \ldots, t_{N_p(0,T)}$ as the switching times on the interval $[0, 1]$, where

$$
N_p(0,T) = \sum_{i}^{M} N_{pi}(0,T). \tag{44}
$$

By integrating (43) for any $t \in [t_i, t_{i+1}]$, we find that

$$
V_{p(t)} \leq e^{\alpha p(t_i)(t-t_i)} V_{p(t)}. \tag{45}
$$

From (18), we can obtain

$$
V_{p(t)} \leq \mu V\left(p_{t_i^-}, p(t_i^-)\right), \quad \forall (p(t_i) = i, p(t_i^-) = j) \in \mathbb{N}\times\mathbb{N}, i \neq j. \tag{46}
$$

By substituting (46) into (45), we can obtain

$$
\begin{aligned}
V_{p(t)} &\le e^{\left\{\alpha_{p\left(t_{N_{p(t)}(0,t)}\right)}\left(t-t_{N_{p(t)}(0,t)}\right)\right\}} V\left(t_{N_p(0,t)}, p\left(t_{N_p(0,t)}\right)\right) \\
&\le \mu_{p\left(t_{N_p}(0,t)\right)} e^{\left\{\alpha_{p\left(t_{N_p}(0,t)\right)}\left(t-t_{N_p}(0,t)\right)\right\}} V\left(x_{t^{-}_{N_p(0,t)}}, p\left(t_{N_p(0,t)-1}\right)\right) \\
&\le \mu_{p\left(t_{N_p}(0,T)\right)} e^{\left\{\alpha_{p\left(t_{N_p}(t,T)\right)}\left(t-t_{N_p(0,T)}\right)+\alpha_{p\left(t_{N_p(0,T)-1}\right)}\left(t_{N_p(0,T)}-t_{N_p(0,T)-1}\right)\right\}} \\
&\quad V\left(x_{t_{N_p(0,T)-1}}, p\left(t_{N_p(0,T)-1}\right)\right) + \mu_{p\left(t_{N_p(0,T)}\right)} \\
&\quad \int_{t_{N_p(0,T)-1}}^{t_{N_p(0,T)}} e^{\left\{\alpha_{p\left(t_{N_p(0,T)}\right)}\left(T-t_{N_p(0,T)}\right)\right\}} + \cdots \\
&\le \prod_{l=0}^{N_p(0,T)-1} \mu_{p(t_{l+1})} e^{\left\{\sum_{l=0}^{N_p(0,T)-1}\left(\alpha_{p(t_{l+1})}+\alpha_{p(t_l)}\right)t_{l+1}+\alpha_{p\left(t_{N_p(0,T)}\right)}^{T+\alpha_{p(t_0)}t_0}\right\}} V\left(x_{t_0}, p(t_0)\right) \\
&\le e^{\left\{\sum_{i=1}^{N}\left(T_i(0,T)/\tau_{ai}\right)\ln\mu_i+\sum_{i=1}^{N}\alpha_i T_i(0,T)\right\}} V\left(x_{t_0}, p(t_0)\right) \\
&\le e^{\left\{\sum_{i=1}^{N}\left(\left(\ln\mu_i/\tau_{ai}\right)+\alpha_i\right)T\right\}} V\left(x_{t_0}, p(t_0)\right).
\end{aligned}
\tag{47}
$$

On the other hand, it follows from (22) that

$$
\begin{aligned}
V_{p(0)}(0) &= x^T(0)P_{1i}x(0) + y^T(0)P_{2i}y(0) + \int_{-\sigma_1}^{0} e^{\alpha(s)}x^T(s)Q_{1i}x(s)\mathrm{d}s + \int_{-\sigma_2}^{-\sigma_1} e^{\alpha(s)}x^T(s)Q_{2i}x(s)\mathrm{d}s \\
&+ \int_{-\sigma(0)}^{0} e^{\alpha(s)}x^T(s)Q_{3i}x(s)\mathrm{d}s + \int_{-\tau_1}^{0} e^{\alpha(s)}y^T(s)R_{1i}y(s)\mathrm{d}s \\
&+ \int_{-\tau_2}^{-\tau_1} e^{\alpha(s)}y^T(s)R_{2i}y(s)\mathrm{d}s + \int_{-\tau(0)}^{0} e^{\alpha(s)}\begin{bmatrix} y(s) \\ g(y(s)) \end{bmatrix}^T \begin{bmatrix} R_{3i} & M_{1i} \\ * & R_{4i} \end{bmatrix}\begin{bmatrix} y(s) \\ g(y(s)) \end{bmatrix}\mathrm{d}s \\
&+ \int_{-\sigma_1}^{0}\int_{\theta}^{0} \sigma_1 e^{\alpha(s)}x^T(s)R_{5i}x(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\sigma_2}^{-\sigma_2}\int_{\theta}^{0} \sigma_{12} e^{\alpha(s)}x^T(s)R_{6i}x(s)\mathrm{d}s\mathrm{d}\theta \\
&+ \int_{-\sigma_1}^{0}\int_{\theta}^{0} \sigma_1 e^{\alpha(s)}\dot{x}^T(s)R_{7i}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\sigma_2}^{-\sigma_2}\int_{\theta}^{0} \sigma_{12} e^{\alpha(s)}\dot{x}^T(s)R_{8i}\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \\
&+ \int_{-\tau_1}^{0}\int_{\theta}^{0} \tau_1 e^{\alpha(s)}y^T(s)R_{9i}y(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau_2}^{-\tau_2}\int_{\theta}^{0} \tau_{12} e^{\alpha(s)}y^T(s)R_{10i}y(s)\mathrm{d}s\mathrm{d}\theta \\
&+ \int_{-\tau_1}^{0}\int_{\theta}^{0} \tau_1 e^{\alpha(s)}\dot{y}^T(s)R_{11i}\dot{y}(s)\mathrm{d}s\mathrm{d}\theta + \int_{-\tau_2}^{-\tau_2}\int_{\theta}^{0} \tau_{12} e^{\alpha(s)}\dot{y}^T(s)R_{12i}\dot{y}(s)\mathrm{d}s\mathrm{d}\theta
\end{aligned}
$$

$$
\begin{aligned}
&+\int_{-\sigma_2}^{0}\int_{\theta}^{0}\int_{\nu}^{0}\frac{\sigma_1^2}{2}e^{\alpha(s-\theta)}\dot{x}^T(s)S_{1i}\dot{x}(s)\mathrm{d}s\mathrm{d}\nu\mathrm{d}\theta+\int_{-\sigma_2}^{-\sigma_1}\int_{\theta}^{0}\int_{\nu}^{0}\sigma_{13}e^{\alpha(s-\theta)}\dot{x}^T(s)S_{2i}\dot{x}(s)\mathrm{d}s\mathrm{d}\nu\mathrm{d}\theta\\
&+\int_{-\tau_2}^{0}\int_{\theta}^{0}\int_{\nu}^{0}\frac{\tau_1^2}{2}e^{\alpha(s-\theta)}\dot{y}^T(s)S_{3i}\dot{y}(s)\mathrm{d}s\mathrm{d}\nu\mathrm{d}\theta+\int_{-\tau_2}^{-\tau_1}\int_{\theta}^{0}\int_{\nu}^{0}\tau_{13}e^{\alpha(s-\theta)}\dot{y}^T(s)S_{4i}\dot{y}(s)\mathrm{d}s\mathrm{d}\nu\mathrm{d}\theta,
\end{aligned}
$$

$$
\begin{aligned}
V_{p(0)}(0)\le\Bigg\{&\max_{i\in\mathbb{N}}\lambda_{\max}(P_{1i})+\max_{i\in\mathbb{N}}\lambda_{\max}(P_{2i})+\sigma_1e^{-\alpha\sigma_1}\max_{i\in\mathbb{N}}\lambda_{\max}(Q_{1i})+(\sigma_2-\sigma_1)e^{-\alpha\sigma_2}\max_{i\in\mathbb{N}}\lambda_{\max}(Q_{2i}\\
&+\sigma_2e^{-\alpha\sigma_2}\max_{i\in\mathbb{N}}\lambda_{\max}(Q_{3i})+\tau_1e^{-\alpha\tau_1}\max_{i\in\mathbb{N}}\lambda_{\max}(R_{1i})+(\tau_2-\tau_1)e^{-\alpha\tau_1}\max_{i\in\mathbb{N}}\lambda_{\max}(R_{2i})\\
&+\tau_2e^{-\alpha\tau_2}\max_{i\in\mathbb{N}}\lambda_{\max}(R_{3i})+2\tau_2e^{-\alpha\tau_2}\max_{i\in\mathbb{N}}\lambda_{\max}(M_{1i})+\tau_2e^{-\alpha\tau_2}\max_{i\in\mathbb{N}}\lambda_{\max}(R_{4i})\left[\max\left(\left|L_i^-,L_i^+\right|\right)\right]^2\\
&+\frac{\sigma_1^3}{2}e^{-\alpha\sigma_1}\max_{i\in\mathbb{N}}\lambda_{\max}(R_{5i})+\frac{(\sigma_2-\sigma_1)^3}{2}e^{-\alpha\sigma_2}\max_{i\in\mathbb{N}}\lambda_{\max}(R_{6i})+\frac{\sigma_1^3}{2}e^{-\alpha\sigma_1}\max_{i\in\mathbb{N}}\lambda_{\max}(R_{7i})\\
&+\frac{(\sigma_2-\sigma_1)^3}{2}e^{-\alpha\sigma_2}\max_{i\in\mathbb{N}}\lambda_{\max}(R_{8i})+\frac{\tau_1^3}{2}e^{-\alpha\tau_1}\max_{i\in\mathbb{N}}\lambda_{\max}(R_{9i})+\frac{(\tau_2-\tau_1)^3}{2}e^{-\alpha\tau_2}\max_{i\in\mathbb{N}}\lambda_{\max}(R_{10i})\\
&+\frac{\tau_1^3}{2}e^{-\alpha\tau_1}\max_{i\in\mathbb{N}}\lambda_{\max}(R_{11i})+\frac{(\tau_2-\tau_1)^3}{2}e^{-\alpha\tau_2}\max_{i\in\mathbb{N}}\lambda_{\max}(R_{12i})+\frac{\sigma_1^5}{4}e^{-\alpha\sigma_1}\max_{i\in\mathbb{N}}\lambda_{\max}(S_{1i})\\
&+\frac{(\sigma_2^2-\sigma_1^2)(\sigma_2-\sigma_1)^3}{4}e^{-\alpha\sigma_2}\max_{i\in\mathbb{N}}\lambda_{\max}(S_{2i})+\frac{\tau_1^5}{4}e^{-\alpha\tau_1}\max_{i\in\mathbb{N}}\lambda_{\max}(S_{3i})\\
&+\frac{(\tau_2^2-\tau_1^2)(\tau_2-\tau_1)^3}{4}e^{-\alpha\tau_2}\max_{i\in\mathbb{N}}\lambda_{\max}(S_{4i})\Bigg\}\\
&\times\sup_{-\rho\le t\le0}\left\{\|\Phi(t)\|^2,\|\Psi(t)\|^2\right\},\\
\le\Big(&\lambda_3+\lambda_4+\lambda_5\sigma_1e^{-\alpha\sigma_1}+\lambda_6(\sigma_2-\sigma_1)e^{-\alpha\sigma_2}+\lambda_7\sigma_2e^{-\alpha\sigma_2}+\lambda_8\tau_1e^{-\alpha\tau_1}\\
&+\lambda_9(\tau_2-\tau_1)e^{-\alpha\tau_1}+\lambda_{10}\tau_2e^{-\alpha\tau_2}+2\lambda_{11}\tau_2e^{-\alpha\tau_2}+\lambda_{12}\tau_2e^{-\alpha\tau_2}+\lambda_{13}\frac{\sigma_1^3}{2}e^{-\alpha\sigma_1}\\
&+\lambda_{14}\frac{(\sigma_2-\sigma_1)^3}{2}e^{-\alpha\sigma_2}+\lambda_{15}\frac{\sigma_1^3}{2}e^{-\alpha\sigma_1}+\lambda_{16}\frac{(\tau_2-\tau_1)^3}{2}e^{-\alpha\tau_2}+\lambda_{17}\frac{\tau_1^3}{2}e^{-\alpha\tau_1}\\
&+\lambda_{18}\frac{(\tau_2-\tau_1)^3}{2}e^{-\alpha\tau_2}+\lambda_{19}\frac{\tau_1^3}{2}e^{-\alpha\tau_1}+\lambda_{20}\frac{(\tau_2-\tau_1)^3}{2}e^{-\alpha\tau_2}+\lambda_{21}\frac{\sigma_1^5}{4}e^{-\alpha\sigma_1}\\
&+\lambda_{22}\frac{(\sigma_2^2-\sigma_1^2)(\sigma_2-\sigma_1)^3}{4}e^{-\alpha\sigma_2}+\lambda_{23}\frac{\tau_1^5}{4}e^{-\alpha\tau_1}+\lambda_{24}\frac{(\tau_2^2-\tau_1^2)(\tau_2-\tau_1)^3}{4}e^{-\alpha\tau_2}\Bigg)c_1,\\
=\,&dc_1.
\end{aligned}
\tag{48}
$$

Then, we can easily obtain

$$V_{p(t)}(t) = e^{\left\{\sum_{i=1}^{N}((\ln\mu_i/\tau_{ai})-\alpha_i)T\right\}}dc_1, \quad (49)$$

$$\begin{aligned} V_{p(t)}(t) &= x^T(t)P_{1i}x(t) + y^T(t)P_{2i}y(t) \\ &\geq \min_{i\in\mathbb{N}}\left[\lambda_{\min}(P_{1i})\|x(t)\|^2 + \lambda_{\min}(P_{2i})\|y(t)\|^2\right] \\ &= (\lambda_1+\lambda_2)\left\{\|x(t)\|^2 + \|y(t)\|^2\right\}. \end{aligned} \quad (50)$$

From (49) and (50), we obtain

$$\|x(t)\|^2 + \|y(t)\|^2 \leq \frac{e^{\left\{\sum_{i=1}^{N}((\ln\mu_i/\tau_{ai})-\alpha_i)T\right\}}dc_1}{(\lambda_1+\lambda_2)}. \quad (51)$$

Therefore, by Definition 1, we conclude that $\|x(t)\|^2 + \|y(t)\|^2 < c_2$. This completes the proof.

Remark 1. We introduce $\int_{-\sigma_2}^{0}\int_{\theta}^{0}\int_{t+\nu}^{t}(\sigma_1^2/2)\; e^{\alpha(s-\theta)}\dot{x}^T(s)S_{1i}\dot{x}(s)\mathrm{d}s\mathrm{d}\nu\mathrm{d}\theta + \int_{-\sigma_2}^{-\sigma_1}\int_{\theta}^{0}\int_{t+\nu}^{t}\sigma_{13}e^{\alpha(s-\theta)}\dot{x}^T(s)S_{2i}\dot{x}(s)\mathrm{d}s\mathrm{d}\nu\mathrm{d}\theta + \int_{-\tau_2}^{0}\int_{\theta}^{0}\int_{t+\nu}^{t}(\tau_1^2/2)e^{\alpha(s-\theta)}\dot{y}^T(s)S_{3i}\dot{y}(s)\mathrm{d}s\mathrm{d}\nu\mathrm{d}\theta + \int_{-\tau_2}^{-\tau_1}\int_{\theta}^{0}\int_{t+\nu}^{t}\tau_{13}e^{\alpha(s-\theta)}\dot{y}^T(s)S_{4i}\dot{y}(s)\mathrm{d}s\mathrm{d}\nu\mathrm{d}\theta$ in our Lyapunov–Krasovskii functional. The novel Lyapunov–Krasovskii functional can make the stability criteria applicable to both fast and slow time-varying delays directly. Besides, by using the convex combination technique together with the Jensen inequality lemma, less conservative criteria are obtained.

Case: we consider the following genetic regulatory networks without switched term:

$$\begin{cases} \dot{x}(t) = -Ax(t) + Bg(y(t-\tau(t))), \\ \dot{y}(t) = -Cy(t) + D(x(t-\sigma(t))). \end{cases} \quad (52)$$

Based on Theorem 1, the next rate-independent corollary is derived.

Corollary 1. *The genetic regulatory network (52) is asymptotically stable with respect to positive real numbers $\sigma_1, \sigma_2, \tau_1, \tau_2$; if there exist symmetric positive definite matrices $P_1, P_2, Q_n (n = 1,2,3)$, $R_n (n = 1,2,\ldots,12)$, $S_n (n = 1,2,\ldots,4)$, the diagonal matrix $L_m = diag(l_{1m}, l_{2m},\ldots,l_{nm}) \geq 0, m = 1,2$ such that the following LMIs hold*

$$\begin{aligned} &\begin{bmatrix} R_3 & M_1 \\ * & R_4 \end{bmatrix} \geq 0, \\ &\begin{bmatrix} R_6 & \overline{R}_6 \\ * & R_6 \end{bmatrix} \geq 0, \\ &\begin{bmatrix} R_8 & \overline{R}_8 \\ * & R_8 \end{bmatrix} \geq 0, \\ &\begin{bmatrix} R_{10} & \overline{R}_{10} \\ * & R_{10} \end{bmatrix} \geq 0, \\ &\begin{bmatrix} R_{12} & \overline{R}_{12} \\ * & R_{12} \end{bmatrix} \geq 0, \\ &\overline{\Theta} = \left[\overline{\Theta}_{ij}\right]_{22\times 22} < 0, \end{aligned} \quad (53)$$

where,

$$\begin{aligned}
\overline{\Theta}_{11} &= -P_1A - A^TP_1^T + Q_1 + Q_3 + \sigma_1^2R_5 + \sigma_{12}^2R_6 - 3e^{-\alpha\sigma_1}R_7 - e^{-\alpha\sigma_1}R_7 + \alpha P_1, \\
\overline{\Theta}_{13} &= e^{-\alpha\sigma_1}R_7 - 3e^{-\alpha\sigma_1}R_7, \\
\overline{\Theta}_{110} &= P_1B, \overline{\Theta}_{113} = \frac{6}{\sigma_1}e^{-\alpha\sigma_1}R_7, \\
\overline{\Theta}_{22} &= R_1 + R_3 - P_2C - C^TP_2^T + \tau_1^2R_9 + \tau_{12}^2R_{10} - e^{-\alpha\tau_1}R_{11} - 3e^{-\alpha\tau_1}R_{11} + \alpha P_2, \\
\overline{\Theta}_{24} &= P_2D, \\
\overline{\Theta}_{26} &= e^{-\alpha\tau_1}R_{11} - 3e^{-\alpha\tau_1}R_{11}, \\
\overline{\Theta}_{29} &= M_1 + U^TL_1^T, \overline{\Theta}_{218} = \frac{6}{\tau_1}e^{-\alpha\tau_1}R_{11}, \\
\overline{\Theta}_{33} &= -e^{-\alpha\sigma_1}Q_1 + e^{-\alpha\sigma_1}Q_2 - e^{-\alpha\sigma_1}R_7 - 3e^{-\alpha\sigma_1}R_7 - e^{-\alpha\sigma_2}R_8, \\
\overline{\Theta}_{34} &= -e^{-\alpha\sigma_2}\overline{R}_8^T + e^{-\alpha\sigma_2}R_8, \\
\overline{\Theta}_{35} &= e^{-\alpha\sigma_2}\overline{R}_8^T, \\
\overline{\Theta}_{313} &= \frac{6}{\sigma_1}e^{-\alpha\sigma_1}R_7,
\end{aligned}$$

$$\begin{aligned}
\overline{\Theta}_{44} &= -(1-\sigma_d)e^{-\alpha\sigma(t)}Q_3 + 2e^{-\alpha\sigma_2}\overline{R}_8 - 2e^{-\alpha\sigma_2}R_8,\\
\overline{\Theta}_{45} &= e^{-\alpha\sigma_2}\overline{R}_8^T + e^{-\alpha\sigma_2}R_8,\\
\overline{\Theta}_{55} &= e^{-\alpha\sigma_2}Q_2^T - e^{-\alpha\sigma_2}R_8,\\
\overline{\Theta}_{66} &= -e^{-\alpha\tau_1}R_1 + e^{-\alpha\tau_1}R_2 - e^{-\alpha\tau_1}R_{11} - 3e^{-\alpha\tau_1}R_{11} - e^{-\alpha\tau_2}R_{12},\\
\overline{\Theta}_{67} &= -e^{-\alpha\tau_2}\overline{R}_{12}^T + e^{-\alpha\tau_2}R_{12},\\
\overline{\Theta}_{68} &= -e^{-\alpha\tau_2}\overline{R}_{12}^T,\\
\overline{\Theta}_{618} &= \frac{6}{\tau_1}e^{-\alpha\tau_1}R_{11},\\
\overline{\Theta}_{77} &= -(1-\tau_d)e^{-\alpha\tau(t)}R_3 + 2e^{-\alpha\tau_2}\overline{R}_{12} - 2e^{-\alpha\tau_2}R_{12},\\
\overline{\Theta}_{78} &= e^{-\alpha\tau_2}\overline{R}_{12}^T + e^{-\alpha\tau_2}R_{12},\\
\overline{\Theta}_{710} &= -(1-\tau_d)e^{-\alpha\tau(t)}M_1 + UL_2,\\
\overline{\Theta}_{88} &= -e^{-\alpha\tau_2}R_2^T - e^{-\alpha\tau_2}R_{12},\\
\overline{\Theta}_{99} &= R_4 - 2L_1,\\
\overline{\Theta}_{1010} &= -(1-\tau_d)e^{-\alpha\tau_2}R_4 - 2L_2,\\
\overline{\Theta}_{1111} &= \sigma_1^2R_7 + \sigma_{12}^2R_8 + e^{\alpha t}\frac{e^{\alpha\sigma_2} - \alpha\sigma_2 - 1}{\alpha^2}S_1 + e^{\alpha t}\frac{e^{\alpha\sigma_2} - e^{\alpha\sigma_1} - \alpha(\sigma_2-\sigma_1)}{\alpha^2}S_2,\\
\overline{\Theta}_{1212} &= \tau_1^2R_{11} + \tau_{12}^2R_{12} + e^{\alpha t}\frac{e^{\alpha\tau_2} - \alpha\tau_2 - 1}{\alpha^2}S_3 + e^{\alpha t}\frac{e^{\alpha\tau_2} - e^{\alpha\tau_1} - \alpha(\tau_2-\tau_1)}{\alpha^2}S_4,\\
\overline{\Theta}_{1313} &= -e^{\alpha\sigma_1}R_5 - \frac{12}{\sigma_1^2}e^{-\alpha\sigma_1}R_7,\\
\overline{\Theta}_{1414} &= -e^{\alpha\sigma_2}R_6,\\
\overline{\Theta}_{1415} &= -e^{\alpha\sigma_2}\overline{R}_6,\\
\overline{\Theta}_{1515} &= -e^{\alpha\sigma_2}R_6,\\
\overline{\Theta}_{1616} &= -e^{\alpha t}S_1,\\
\overline{\Theta}_{1717} &= -e^{\alpha t}S_2,\\
\overline{\Theta}_{1818} &= -e^{\alpha\tau_1}R_9 - \frac{12}{\tau_1^2}e^{-\alpha\tau_1}R_{11},\\
\overline{\Theta}_{1919} &= -e^{\alpha\tau_2}R_{10},\\
\overline{\Theta}_{1920} &= -e^{\alpha\tau_2}\overline{R}_{10},\\
\overline{\Theta}_{2020} &= -e^{\alpha\tau_2}R_{10},\\
\overline{\Theta}_{2121} &= -e^{\alpha t}S_3,\\
\overline{\Theta}_{2222} &= -e^{\alpha t}S_4.
\end{aligned} \tag{54}$$

Proof. The corollary follows by a similar argument as that in proof of Theorem 1.

4. Numerical Example

In this section, numerical examples are provided to illustrate the validity and the advantage of the proposed finite-time stability of switched GRNs with time-varying delays.

Example 1. Consider switched GRNs with time-varying delay (8) as

$$\begin{cases} \dot{x}(t) = -A_{p(t)}x(t) + B_{p(t)}g(y(t-\tau(t))), \\ \dot{y}(t) = -C_{p(t)}y(t) + D_{p(t)}(x(t-\sigma(t))), \end{cases} \tag{55}$$

with

$$\begin{gathered} A_1 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \\ B_1 = \begin{bmatrix} 0 & 0 & -2 \\ -2 & 0 & 0 \\ 0 & -2 & 0 \end{bmatrix}, \\ C_1 = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \\ D_1 = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.9 \end{bmatrix}, \\ A_2 = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \\ B_2 = \begin{bmatrix} 0 & -1.5 & 0 \\ -1.5 & 0 & 0 \\ 0 & -1.5 & 0 \end{bmatrix}, \\ C_2 = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}, \\ D_2 = \begin{bmatrix} 1.2 & 0 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 1.2 \end{bmatrix}. \end{gathered} \tag{56}$$

The activation function is chosen as $U = \mathrm{diag}\{0.65, 0.65, 0.65\}$, and the values of c_1, c_2, T are given as follows:

$$\begin{aligned} \sigma_1 &= 0.7, \\ \sigma_2 &= 3.5, \\ \sigma_d &= 0.4, \\ \tau_1 &= 0.6, \\ \tau_2 &= 3.2, \\ \tau_d &= 0.2, \\ c_1 &= 1.5, \\ c_2 &= 4.5, \\ T &= 6, \\ \mu &= 1.5. \end{aligned} \tag{57}$$

We show the simulation result of the trajectories of the variables $x(t)$ and $y(t)$ in Figures 1 and 2. It should be point out that the condition is feasible when employing the LMI toolbox in MATLAB, solve LMIs (16)–(20), and then, we can reach feasible solution. Hence, the switched GRNs (8) is finite-time stable.

$$\begin{gathered} P_{11} = \begin{bmatrix} 0.1837 & -0.0007 & -0.0007 \\ -0.0007 & 0.1837 & -0.0007 \\ -0.0007 & -0.0007 & 0.1837 \end{bmatrix}, \\ P_{21} = \begin{bmatrix} 0.5336 & 0.0008 & 0.0008 \\ 0.0008 & 0.5336 & 0.0008 \\ 0.0008 & 0.0008 & 0.5336 \end{bmatrix}, \\ Q_{11} = \begin{bmatrix} 0.0419 & -0.0000 & -0.0000 \\ -0.0000 & 0.0419 & -0.0000 \\ -0.0000 & -0.0000 & 0.0419 \end{bmatrix}, \\ Q_{21} = \begin{bmatrix} 0.0225 & 0.0000 & 0.0000 \\ 0.0000 & 0.0225 & 0.0000 \\ 0.0000 & 0.0000 & 0.0225 \end{bmatrix}, \\ Q_{31} = \begin{bmatrix} 0.1556 & 0.0003 & 0.0003 \\ 0.0003 & 0.1556 & 0.0003 \\ 0.0003 & 0.0003 & 0.1556 \end{bmatrix}, \\ R_{11} = \begin{bmatrix} 0.0669 & 0.0001 & 0.0001 \\ 0.0001 & 0.0669 & 0.0001 \\ 0.0001 & 0.0001 & 0.0669 \end{bmatrix}, \\ R_{21} = \begin{bmatrix} 0.0287 & 0.0000 & 0.0000 \\ 0.0000 & 0.0287 & 0.0000 \\ 0.0000 & 0.0000 & 0.0287 \end{bmatrix}, \\ R_{31} = \begin{bmatrix} 0.1518 & -0.0001 & -0.0001 \\ -0.0001 & 0.1518 & -0.0001 \\ -0.0001 & -0.0001 & 0.1518 \end{bmatrix}, \end{gathered}$$

$$R_{41} = \begin{bmatrix} 0.0389 & 0.0000 & 0.0000 \\ 0.0000 & 0.0389 & 0.0000 \\ 0.0000 & 0.0000 & 0.0389 \end{bmatrix},$$

$$R_{51} = \begin{bmatrix} 0.0570 & -0.0000 & -0.0000 \\ -0.0000 & 0.0570 & -0.0000 \\ -0.0000 & -0.0000 & 0.0570 \end{bmatrix},$$

$$R_{61} = \begin{bmatrix} 0.0053 & 0.0000 & 0.0000 \\ 0.0000 & 0.0053 & 0.0000 \\ 0.0000 & 0.0000 & 0.0053 \end{bmatrix},$$

$$R_{71} = \begin{bmatrix} 0.0562 & 0.0000 & 0.0000 \\ 0.0000 & 0.0562 & 0.0000 \\ 0.0000 & 0.0000 & 0.0562 \end{bmatrix},$$

$$R_{81} = \begin{bmatrix} 0.0057 & -0.0000 & -0.0000 \\ -0.0000 & 0.0057 & -0.0000 \\ -0.0000 & -0.0000 & 0.0057 \end{bmatrix},$$

$$R_{91} = \begin{bmatrix} 0.1006 & 0.0000 & 0.0000 \\ 0.0000 & 0.1006 & 0.0000 \\ 0.0000 & 0.0000 & 0.1006 \end{bmatrix},$$

$$R_{101} = \begin{bmatrix} 0.0094 & 0.0000 & 0.0000 \\ 0.0000 & 0.0094 & 0.0000 \\ 0.0000 & 0.0000 & 0.0094 \end{bmatrix},$$

$$R_{111} = \begin{bmatrix} 0.0627 & -0.0001 & -0.0001 \\ -0.0001 & 0.0627 & -0.0001 \\ -0.0001 & -0.0001 & 0.0627 \end{bmatrix},$$

$$R_{121} = \begin{bmatrix} 0.0057 & -0.0000 & -0.0000 \\ -0.0000 & 0.0057 & -0.0000 \\ -0.0000 & -0.0000 & 0.0057 \end{bmatrix},$$

$$S_{11} = \begin{bmatrix} 0.0016 & -0.0000 & -0.0000 \\ -0.0000 & 0.0016 & -0.0000 \\ -0.0000 & -0.0000 & 0.0016 \end{bmatrix},$$

$$S_{21} = 10^{-3} \times \begin{bmatrix} 0.2915 & 0.0005 & 0.0005 \\ 0.0005 & 0.2915 & 0.0005 \\ 0.0005 & 0.0005 & 0.2915 \end{bmatrix},$$

$$S_{31} = \begin{bmatrix} 0.0023 & 0.0000 & 0.0000 \\ 0.0000 & 0.0023 & 0.0000 \\ 0.0000 & 0.0000 & 0.0023 \end{bmatrix},$$

$$S_{41} = 10^{-3} \times \begin{bmatrix} 0.4960 & 0.0007 & 0.0007 \\ 0.0007 & 0.4960 & 0.0007 \\ 0.0007 & 0.0007 & 0.4960 \end{bmatrix},$$

$$P_{12} = \begin{bmatrix} 0.1747 & 0.0001 & 0.0001 \\ 0.0001 & 0.1747 & 0.0001 \\ 0.0001 & 0.0001 & 0.1747 \end{bmatrix},$$

$$P_{22} = \begin{bmatrix} 0.1267 & 0.0003 & 0.0003 \\ 0.0003 & 0.1267 & 0.0003 \\ 0.0003 & 0.0003 & 0.1267 \end{bmatrix},$$

$$Q_{12} = \begin{bmatrix} 0.1265 & 0.0004 & 0.0004 \\ 0.0004 & 0.1265 & 0.0004 \\ 0.0004 & 0.0004 & 0.1265 \end{bmatrix},$$

$$Q_{22} = \begin{bmatrix} 0.1121 & -0.0004 & -0.0004 \\ -0.0004 & 0.1121 & -0.0004 \\ -0.0004 & -0.0004 & 0.1121 \end{bmatrix},$$

$$Q_{32} = \begin{bmatrix} 0.1745 & 0.0014 & 0.0014 \\ 0.0014 & 0.1745 & 0.0014 \\ 0.0014 & 0.0014 & 0.1745 \end{bmatrix},$$

$$R_{12}=\begin{bmatrix}0.1231 & 0.0008 & 0.0008\\ 0.0008 & 0.1231 & 0.0008\\ 0.0008 & 0.0008 & 0.1231\end{bmatrix},$$

$$R_{22}=\begin{bmatrix}0.1145 & -0.0005 & -0.0005\\ -0.0005 & 0.1145 & -0.0005\\ -0.0005 & -0.0005 & 0.1145\end{bmatrix},$$

$$R_{32}=\begin{bmatrix}0.1434 & 0.0020 & 0.0020\\ 0.0020 & 0.1434 & 0.0020\\ 0.0020 & 0.0020 & 0.1434\end{bmatrix},$$

$$R_{42}=\begin{bmatrix}0.0863 & 0.0001 & 0.0001\\ 0.0001 & 0.0863 & 0.0001\\ 0.0001 & 0.0001 & 0.0863\end{bmatrix},$$

$$R_{52}=\begin{bmatrix}0.1176 & -0.0002 & -0.0002\\ -0.0002 & 0.1176 & -0.0002\\ -0.0002 & -0.0002 & 0.1176\end{bmatrix},$$

$$R_{62}=\begin{bmatrix}0.0641 & -0.0005 & -0.0005\\ -0.0005 & 0.0641 & -0.0005\\ -0.0005 & -0.0005 & 0.0641\end{bmatrix},$$

$$R_{72}=\begin{bmatrix}0.0886 & -0.0005 & -0.0005\\ -0.0005 & 0.0886 & -0.0005\\ -0.0005 & -0.0005 & 0.0886\end{bmatrix},$$

$$R_{82}=\begin{bmatrix}0.1177 & -0.0004 & -0.0004\\ -0.0004 & 0.1177 & -0.0004\\ -0.0004 & -0.0004 & 0.1177\end{bmatrix},$$

$$R_{92}=\begin{bmatrix}0.1230 & -0.0001 & -0.0001\\ -0.0001 & 0.1230 & -0.0001\\ -0.0001 & -0.0001 & 0.1230\end{bmatrix},$$

$$R_{102}=\begin{bmatrix}0.0675 & -0.0005 & -0.0005\\ -0.0005 & 0.0675 & -0.0005\\ -0.0005 & -0.0005 & 0.0675\end{bmatrix},$$

$$R_{112}=\begin{bmatrix}0.0878 & -0.0006 & -0.0006\\ -0.0006 & 0.0878 & -0.0006\\ -0.0006 & -0.0006 & 0.0878\end{bmatrix},$$

$$R_{122}=\begin{bmatrix}0.1055 & -0.0008 & -0.0008\\ -0.0008 & 0.1055 & -0.0008\\ -0.0008 & -0.0008 & 0.1055\end{bmatrix},$$

$$S_{12}=\begin{bmatrix}0.1889 & -0.0000 & -0.0000\\ -0.0000 & 0.1889 & -0.0000\\ -0.0000 & -0.0000 & 0.1889\end{bmatrix},$$

$$S_{22}=\begin{bmatrix}0.0736 & 0.0000 & 0.0000\\ 0.0000 & 0.0736 & 0.0000\\ 0.0000 & 0.0000 & 0.0736\end{bmatrix},$$

$$S_{32}=\begin{bmatrix}0.1937 & -0.0000 & -0.0000\\ -0.0000 & 0.1937 & -0.0000\\ -0.0000 & -0.0000 & 0.1937\end{bmatrix},$$

$$S_{42}=\begin{bmatrix}0.0685 & 0.0000 & 0.0000\\ 0.0000 & 0.0685 & 0.0000\\ 0.0000 & 0.0000 & 0.0685\end{bmatrix}. \tag{58}$$

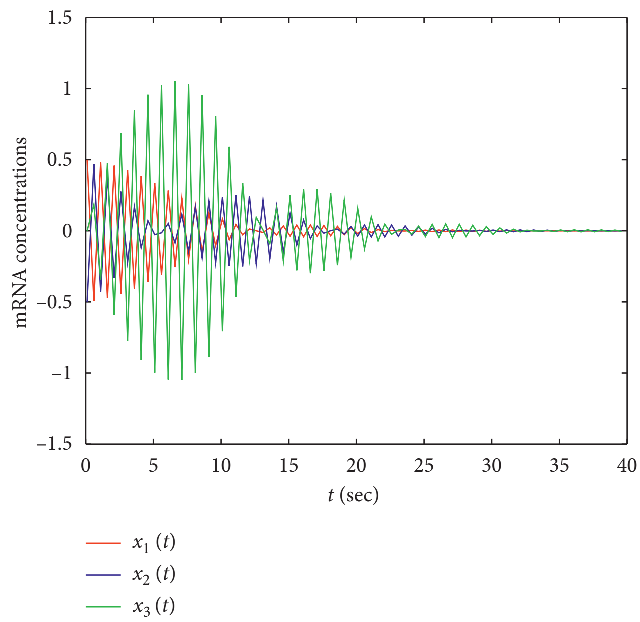

Figure 1: The mRNA concentrations $x(t)$ in Example 1.

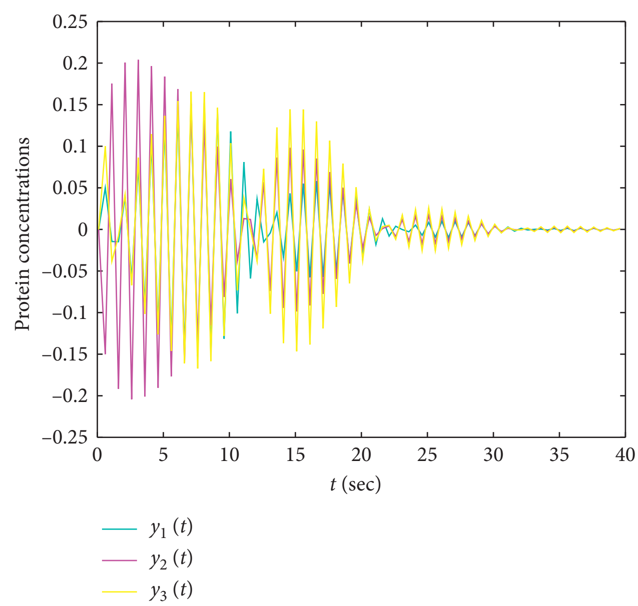

Figure 2: Protein concentrations $y(t)$ in Example 2.

Table 1: The maximum allowable time delay upper bound for τ_2 with different values of σ_{12}.

σ_{12}	0.125	0.25	0.55	1.0	1.1
[15]	0.5	—	—	−8	—
[16]	—	—	1.0	—	—
[17]	2.8273	2.1661	1.1544	0.4904	0.3845
[18]	3.2957	3.1932	2.9455	2.5661	2.4799
Corollary 3.3	5.4924	5.0257	4.6412	4.4670	3.9249

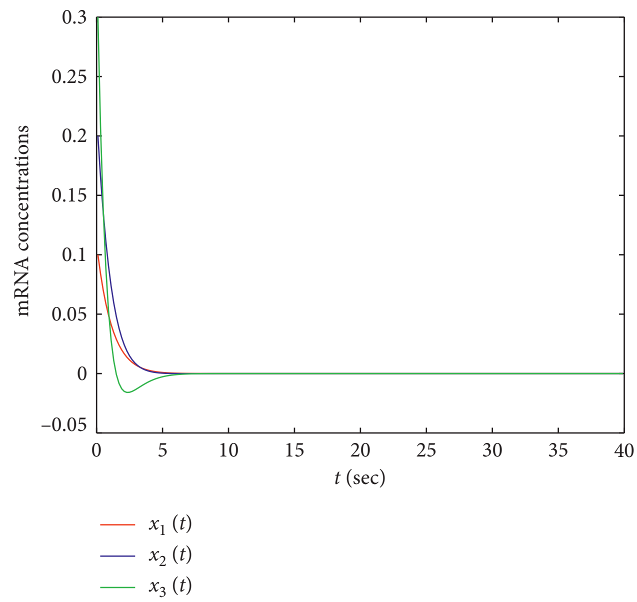

Figure 3: The trajectories of $x(t)$ in Example 2.

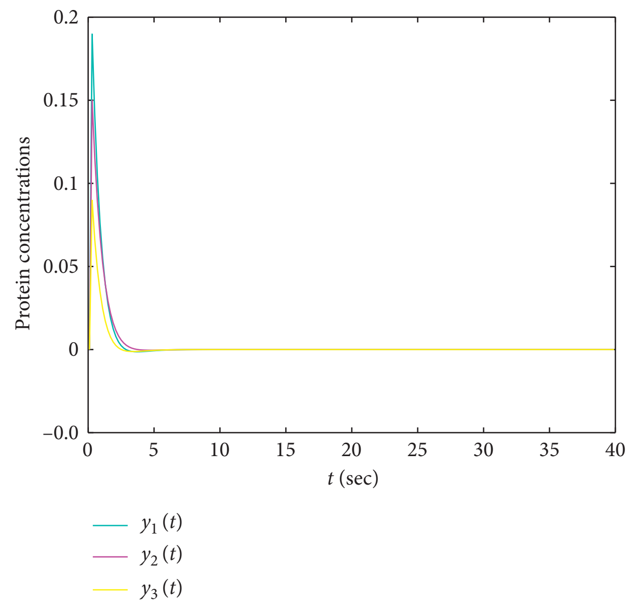

Figure 4: The trajectories of $y(t)$ in Example 2.

Example 2

$$
\begin{aligned}
A &= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \\
B &= \begin{bmatrix} 0 & 0 & -2.5 \\ -2.5 & 0 & 0 \\ 0 & -2.5 & 0 \end{bmatrix}, \\
C &= \begin{bmatrix} 2.5 & 0 & 0 \\ 0 & 2.5 & 0 \\ 0 & 0 & 2.5 \end{bmatrix}, \\
D &= \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 0.8 & 0 \\ 0 & 0 & 0.8 \end{bmatrix},
\end{aligned}
\tag{59}
$$

$g(y) = y^2/1 + y^2$ and $U = \mathrm{diag}\{0.65, 0.65, 0.65\}$.

Furthermore, for the parameters listed above, let $\tau_d = 0.5$ and $\sigma_d = 0.5$.

In order to compare the results in [15–18], using Corollary 1, the comparison results are listed in Table 1 for τ_2. Clearly, the results proposed in this study provide a larger admissible upper bound delay to guarantee the asymptotically stable system (52). In addition, the trajectories of the genetic regulatory network (52) are shown in Figures 3 and 4.

Remark 2. The discussion in Example 2 illustrates that the conditions in this study (Corollary 1) is less conservative than those in [15–18], which shows the superiority of our method compared with that in [15–18].

Example 3. Consider the following switched GRNs with time-varying delay:

$$
\begin{cases} \dot{x}(t) = -A_{p(t)}x(t) + B_{p(t)}g(y(t-\tau(t))), \\ \dot{y}(t) = -C_{p(t)}y(t) + D_{p(t)}(x(t-\sigma(t))), \end{cases}
\tag{60}
$$

with

$$
\begin{aligned}
A_1 &= \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \\
B_1 &= \begin{bmatrix} 0.81 & -0.20 \\ 0.10 & 0.64 \end{bmatrix}, \\
C_1 &= \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \\
D_1 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, \\
B_2 &= \begin{bmatrix} 0.1 & -1 \\ -1 & 0.1 \end{bmatrix}, \\
C_2 &= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, \\
D_2 &= \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}.
\end{aligned}
\tag{61}
$$

The activation function is chosen as $U = \mathrm{diag}\{0.2, 0.2\}$, and the values of c_1, c_2, T are given as follows:

$$\begin{aligned}
\sigma_1 &= 0.2,\\
\sigma_2 &= 3.2,\\
\sigma_d &= 1,\\
\tau_1 &= 0.1,\\
\tau_2 &= 0.3,\\
\tau_d &= 0.1,\\
c_1 &= 1,\\
c_2 &= 3.2,\\
T &= 4,\\
\mu &= 0.9.
\end{aligned} \tag{62}$$

By employing the LMI toolbox in MATLAB, solve LMIs (16)–(20), and the feasible solutions are then reached.

5. Conclusion

In this study, a finite-time stability analysis for switched GRNs with time-varying delays has been investigated. We utilized the reciprocally convex combination method, Wirtinger's integral inequality, and new triple integral with exponential function in Lyapunov–Krasovskii functionals; a less conservative LMI-based finite-time stability criterion is obtained with the switched ADT approach to reduce the conservatism of our results, compared with existing ones. A numerical example has been given to demonstrate the effectiveness and the advantage of our proposed methods. State estimation as well as other research topics such as switched lure systems and complex networks [55, 56] and stabilization of probabilistic Boolean networks [57, 58] of the time delay systems will be further investigated based on the methods proposed in this study.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Authors' Contributions

B.H. involved in funding acquisition; G.R. conceptualized the study, wrote the original draft, and validated; G.R., B.H., M.S.A., and S.S. developed software; G.R. and B.H. performed formal analysis, developed methodology, and reviewed and edited the manuscript; S.S., B.P., G.K.T., and M.S.A. supervised the study. All authors have read and agreed to the published version of the manuscript.

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